

# HW-Section1

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## 1 Homework for Section 1

### 1.1 Simple Discrete Random Variable

Suppose the distribution of the random variable  $X$  is given by the following table

$x$	$p(x)$
---	---
.02	.1
.04	.3
.07	.4
.10	??

(a)

What is  $P(X = .1)$  ?

(b)

What is  $P(X > .05)$ ?

**Solution**

(a) .2, the probabilities have to sum to 1.

(b)  $.4 + .2 = .6$ .

## 1.2 Bernoulli from Die

Suppose you are about to roll a single die a single time.

Let  $B$  be the random variable which is 1 if  $D$  turns out to be greater than 4 and 0 otherwise.

What is the distribution of  $B$ ?

**Solution**

$B \sim \text{Bernoulli}(1/3)$

### 1.3 Disease Testing

Suppose a person is randomly drawn from a large population and then tested for a disease.

Let  $D = 1$  if the person has the disease and 0 otherwise.

Let  $T = 1$  if the person tests positive and 0 otherwise.

Suppose

$$P(D = 0) = .99.$$

$$P(T = 1 \mid D = 0) = .01.$$

$$P(T = 1 \mid D = 1) = .97.$$

(a)

Draw the diagram depicting the marginal of  $D$  and the conditional of  $T \mid D$ .  
(you know, the one that branches as you go left to right).

(b)

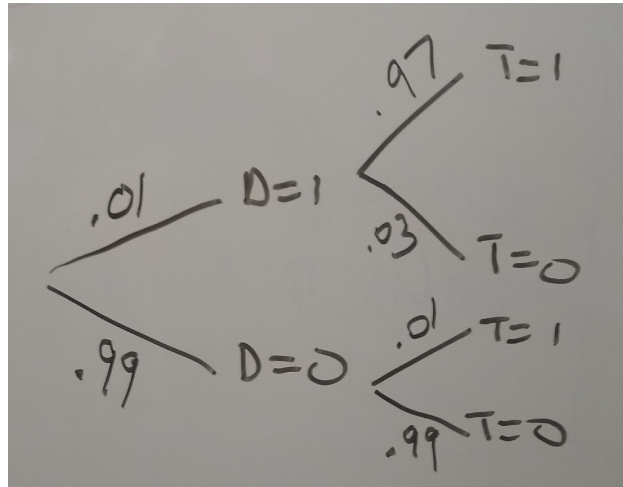
Give the joint distribution of  $D$  and  $T$  in the two way table format.

(c)

What is  $P(D = 1 \mid T = 1)$ ?

## Solution

(a)



(b)

(c)

```
      T0      T1
D0 0.9801 0.0099
D1 0.0003 0.0097
```

```
> .99*.99
[1] 0.9801
> .99*.01
[1] 0.0099
> .01*.03
[1] 3e-04
> .01*.97
[1] 0.0097
```

(c)

$P(D = 1 | T = 1) = P(D = 1, T = 1) / P(T = 1) = .0097 / (.0097 + .0099) = 0.494898.$

## 1.4 Classifying Defaulters

Let's suppose a Credit Card company defines 4 "risk classes" based on the outstanding balance on the card.

risk class 1: balance in  $(0,500]$

risk class 2: balance in  $(500,1000]$

risk class 3: balance in  $(1000,1500]$

risk class 4: balance in  $(1500,\infty)$  (that is, anything  $> 1500$ )

Let  $C$  denote the random variable denoting the risk class of an account and  $D$  denote the random Variable which is 1 if there is a default and 0 if not. The research department has come up with the conditional distribution of  $C$  for  $D=0$  or 1.

	$p(c D=0)$	$p(c D=1)$
1	0.27	0.00
2	0.39	0.02
3	0.27	0.21
4	0.07	0.77

(a)

Suppose  $P(D = 1) = .03$ .

Compute  $P(D = 1|C = c)$  for  $c = 1, 2, 3, 4$ .

(b)

Suppose  $P(D = 1) = .5$ .

Compute  $P(D = 1|C = c)$  for  $c = 1, 2, 3, 4$ .

## Solution

(a)

If  $P(D = 1) = .03$  then the joint distribution of  $(C, D)$  is given by the table:

	d=0	d=1
c=1	0.2619	0.0000
c=2	0.3783	0.0006
c=3	0.2619	0.0063
c=4	0.0679	0.0231

For example  $p(C = 4, D = 0) = p(D = 0)p(C = 4|D = 0) = .97 * .07 = 0.0679$ .

To get the conditional probs  $p(D = 1|c)$  divide the second column by the sum of the two columns:

	c=1	c=2	c=3	c=4
	0.000000000	0.001583531	0.023489933	0.253846154

For example,  $p(D = 1|C = 2) = .0006/ (.0006 + .3783) = 0.001583531$ .

(b)

If  $P(D = 1) = .5$  then the joint distribution of  $(C, D)$  is given by the table:

	d=0	d=1
c=1	0.135	0.000
c=2	0.195	0.010
c=3	0.135	0.105
c=4	0.035	0.385

For example  $.01 = .5 * .02$ ,  $P(C = 2, D = 1) = P(D = 1) * P(C = 2|D = 1)$ .

And the conditional  $P(D = 1|c)$  is:

	c=1	c=2	c=3	c=4
	0.00000000	0.04878049	0.43750000	0.91666667

For example,  $.01/ (.01+.195) = 0.04878049$ .

The marginal  $P(D = 1)$  is called the *prior probability* and the conditional  $P(D = 1|c)$  is called the *posterior probability*.

Clearly, the prior makes a difference!!!



## 1.5 Making Two Parts

You are about to make two parts.

Let  $X = 1$  if the first part is defective and 0 otherwise.

Let  $Y = 1$  if the second part is defective and 0 otherwise.

The table below gives the joint probability distribution for the two random variables  $X$  and  $Y$ .

	X0	X1
Y0	0.81	0.09
Y1	0.09	0.01

- (a) What is the marginal distribution of  $X$ ?
- (b) What is the marginal distribution of  $Y$ ?
- (c) What is the conditional distribution of  $Y \mid X = 1$ ?
- (d) What is the conditional distribution of  $Y \mid X = 0$ ?
- (e) Are  $X$  and  $Y$  independent?
- (f) Are  $X$  and  $Y$  IID?

**Solution**

(a)  $P(X = 0) = .9, P(X = 1) = .1$ .

Or,  $X \sim \text{Bernoulli}(.1)$ .

Or,

x	p(x)
0	.9
1	.1

(b) Same as  $X$ .

(c) Same as marginal for  $Y$ .

(d) Same as marginal for  $Y$ .

(e) They are independent because all conditionals are the same as the marginal.

(f) Yes, because they are independent and have the same distribution.

## 1.6 Democrats and Republicans, Two Draws

Suppose we have 5 Democrats and 5 Republicans.

We are about to sample two voters.

Let  $Y_1$  be 1 if the first voter is a dem and 0 else.

Let  $Y_2$  be 1 if the second voter is a dem and 0 else.

(a)

Suppose we sample without replacement.

Give the 2x2 table representing the joint distribution of  $Y_1$  and  $Y_2$ .

(b)

In (a), are the two  $Y$ 's independent?

Are they identically distributed?

Are they IID?

(c)

Now suppose we sample the two voters with replacement.

Give the 2x2 table representing the joint distribution of  $Y_1$  and  $Y_2$ .

(d)

In (c), are the two  $Y$ 's independent?

Are they identically distributed?

Are they IID?

(e)

Suppose we had 5 million of each instead of just 5.

How would your answers to (a)-(d) change?

## Solution

(a)

	Y1=0	Y1=1
Y2=0	0.222	0.278
Y2=1	0.278	0.222

note:  $(5/10)(4/9) = .222$  (after rounding).  
 $(5/10)(5/9) = .278$  (after rounding).

(b)

They are not independent.  
They are identically distributed.  
They are not IID.

(c)

	Y1=0	Y1=1
Y2=0	0.25	0.25
Y2=1	0.25	0.25

$P(Y1=1, Y2=1) = P(Y1=1) P(Y2=1 | Y1=1) = (5/10) * (5/10) = .25$ .

(d) Yes, they are iid.

(e)

With 5 million, Y1 and Y2 are *approximately* IID even if we sample without replacement.

## 1.7 Monte Hall

This problem is named after the host of the long running TV show *Let's make a deal*. There has been a vigorous debate about what the correct answer is!!

A contestant must choose one of three closed doors. There is a prize (say a car) behind one of the three doors. Behind the other two doors, there is something worthless (traditionally a goat). After the contestant chooses one of the three doors, Monty opens one of the other two, revealing a goat (never the car!!). There are now two closed doors. The contestant is asked whether he would like to switch from the door he initially chose, to the other closed door.

The contestant will get whatever is behind the door that was finally chosen.

Should the contestant switch?

## Solution

Assume (I claim without loss of generality) that you initially select door 1.

The game is about to be played.

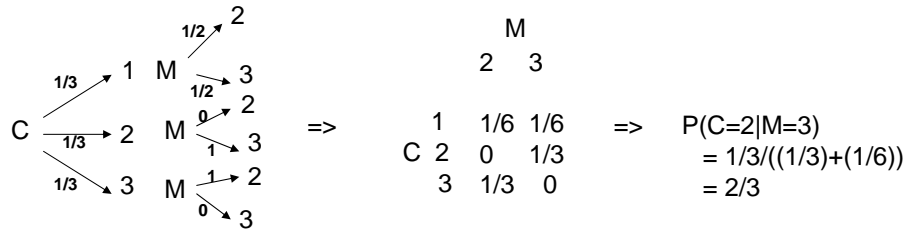
Up to the point where you must decide whether or not to switch, there are two things about which we are uncertain:

C: the door the car is behind, C is 1, 2, or 3

M: which door Monte will open, M is 2 or 3 (given you have selected door 1).

We need the joint distribution of (C,M).

We will first write it in terms of the marginal for C and the conditional for M given C since it is the most obvious in this form.



## 1.8 Spam Filter

Here's a simplified look at a spam filter algorithm.

We are worried about the term *fantastic offer* and our IT team has figured that  $P(\text{fantastic offer}|\text{junk mail}) = 0.20$  and  $P(\text{fantastic offer}|\text{NOT junk mail}) = 0.001$ .

In addition they figured that half of our emails are junk.

Note that  $P(\text{fantastic offer}|\text{junk mail})$  means

“for a email message, given it is junk, what is the probability that the term *fantastic offer* is in it”?

(a)

What is the marginal probability of seeing *fantastic offer* in a message?

In other words, what is the  $P(\text{fantastic offer})$ ?

(b)

If the spam filter always classify a message containing *fantastic offer* as junk, how often will it make a mistake?

In other words, what is the  $P(\text{NOT junk mail}|\text{fantastic offer})$  ?

### Solution

Let's first build the joint distribution table:

	Junk	Not Junk
fantastic offer	$0.2 \times 0.5$	$0.001 \times 0.5$
Not FO	$0.8 \times 0.5$	$0.999 \times 0.5$

	Junk	Not Junk
fantastic offer	0.1	0.0005
Not FO	0.4	0.4995

(a) Therefore, the  $pr(\text{"fantastic offer"}) = 0.1 + 0.0005 = 0.1005$

(b)  $pr(\text{NOT junk mail} | \text{"fantastic offer"}) = \frac{pr(\text{NOT junk mail and "fantastic offer"})}{pr(\text{"fantastic offer"})} \setminus$   
 $pr(\text{NOT junk mail} | \text{"fantastic offer"}) = \frac{0.0005}{0.1005} = 0.005$

i.e., only 0.5% of the emails will be wrongly flagged as junk.



## 1.9 Mean and Variance of A Discrete Random Variable

Suppose the distribution of the random variable  $X$  is given by the following table

$x$	$p(x)$
.02	.1
.04	.3
.07	.4
.10	.2

- (a) What is  $E(X)$ ?
- (b) What is  $Var(X)$ ?

## Solution

```
x = c(.02, .04, .07, .10)
px = c(.1, .3, .4, .2)
M = sum(x*px)
V = sum(px * (x-M)^2)
cat("(a)", M, "\n")
```

```
## (a) 0.062
```

```
cat("(b)", V, "\n")
```

```
## (b) 0.000636
```

## 1.10 Target Marketing

Let's consider a Target Marketing example.

We are trying to decide whether or not to send out a promotion to a particular customer. It costs .8 dollars (80 cents) to do the promotion.

If the customer responds with a purchase you will get 40 dollars.

So our payoff is 39.20 if the target consumer responds and -.8 if they do not.

Let  $p$  be the probability they respond.

Clearly, for a "large"  $p$  we would do the promotion and for a small one we would not.

Here is our random variable  $R$  for revenue (if we do the promotion, with no promotion we get 0 for sure).

$r$	$P(R = r)$
-0.8	$1-p$
39.20	$p$

(a)

Suppose your predictive analytics teams tells you that  $p = .05$ .

What is  $E(R)$ . What is  $\text{Var}(R)$ ? What is  $\sigma_R$ ?

Should you do the promotion ?

(b)

For what value of  $p$  is  $E(R) = 0$  ?

Note that we can write the random variable  $R$  as

$$R = -.8 + 40B$$

where  $B \sim \text{Bernoulli}(p)$ .

For a linear function of a a single random variable we have:

$$E(a + bX) = a + bE(X), \quad \text{Var}(a + bX) = b^2 \text{Var}(X),$$

where  $a$  and  $b$  are constants. This ia simple form of the formulas we discussed for linear combinations of random variables.

(c)

Using the value of  $p$  from part (a) and the formulas for a linear function above, what are  $E(R)$  and  $\text{Var}(R)$  ? Of course you should get the same thing as in part (a) !!

## Solution

(a)

```
rv = c(-.8,39.20)
pv = c(.95,.05)
ER = sum(rv*pv)
```

```
cat("ER is ",ER,"\n")
```

```
## ER is 1.2
```

```
VR = sum(pv*(rv-ER)^2)
```

```
sR = sqrt(VR)
```

```
cat('variance and standard deviation are', VR, ' and ', sR, '\n')
```

```
## variance and standard deviation are 76 and 8.717798
```

Going by expected revenue, you should do the promotion.

(b)

\$  $(-.8) + p, (39.2) = 0$  \$

\$  $p, 40 = .8$ .  $p = \frac{.8}{40} = .02$ .

```
.8/40
```

```
## [1] 0.02
```

(c)

Recall that

$$B \sim \text{Bernoulli}(p) \Rightarrow E(B) = p, \text{Var}(B) = p * (1 - p).$$

```
# mean
```

```
-.8 + .05*40
```

```
## [1] 1.2
```

```
# variance
```

```
(.05)*(.95) * 40^2
```

```
## [1] 76
```

## 1.11 Portfolio of $X$ and $Y$

In our first example of covariance we considered a pair of random variables  $X$  and  $Y$  with joint distribution

x	y	P(x,y)
0.05	0.05	0.4
0.15	0.05	0.1
0.05	0.15	0.1
0.15	0.15	0.4

or,

		X	
		.05	.15
Y	.05	.4	.1
	.15	.1	.4

- (a) Check that  $\text{Var}(X) = \text{Var}(Y) = .0025$ .  
 (b) We showed in the notes that  $\text{Cov}(X, Y) = .0015$ .

Let  $P = .5X + .5Y$  be the portfolios which puts half your money into  $X$  and half your money into  $Y$ .

Using the formulas for the mean and variance of a linear combination, what are  $E(P)$  and  $\text{Var}(P)$ ?

- (c) What is the marginal distribution of  $P$ ?  
 (d) Using the formulas for the mean and variance of a discrete random variable, what are  $E(P)$  and  $\text{Var}(P)$ ?  
 Make sure you get the same thing as in (b)!!  
 (e) Now let  $U$  and  $V$  be random returns with our second joint distribution:

u	v	P(u,v)
0.05	0.05	0.1
0.15	0.05	0.4
0.05	0.15	0.4
0.15	0.15	0.1

and let  $Q = .5U + .5V$ .

$X, Y, U,$  and  $V$  all have the same marginal distributions!!

In the notes we computed  $\text{Cov}(U, V) = -.0015$ .

Using the formulas for the mean and variance of a linear combination, what are  $E(Q)$  and  $\text{Var}(Q)$ ?

- (f) Obtain the marginal distribution of  $Q$ . Plot the distributions of  $P$  and  $Q$ .

Why are the variances so different??!!

## Solution

(a)

$X = .05 + .1 * B$ ,  $B \sim \text{Bernoulli}(.5)$

```
cat("E,V = ", .05 + .1*.5, .1^2 * .5^2,"\n")
```

```
## E,V = 0.1 0.0025
```

(b)

```
EP = .5*.1 + .5*.1
VP = .5^2 * .0025 + .5^2 * .0025 + 2*.5*.5*.0015
cat("EP,VP: ",EP,VP,"\n")
```

```
## EP,VP: 0.1 0.002
```

(c)

p: .05, .1, .15  
p(p): .4, .2, .4

(d)

```
xv = c(.05,.1,.15)
pv1 = c(.4,.2,.4)
M1 = sum(xv*pv1)
V1 = sum(pv1 * (xv-M1)^2)
cat("Mean and var: ",M1,V1,"\n")
```

```
## Mean and var: 0.1 0.002
```

(e)

```
EP = .5*.1 + .5*.1
VP = .5^2 * .0025 + .5^2 * .0025 - 2*.5*.5*.0015
cat("EP,VP: ",EP,VP,"\n")
```

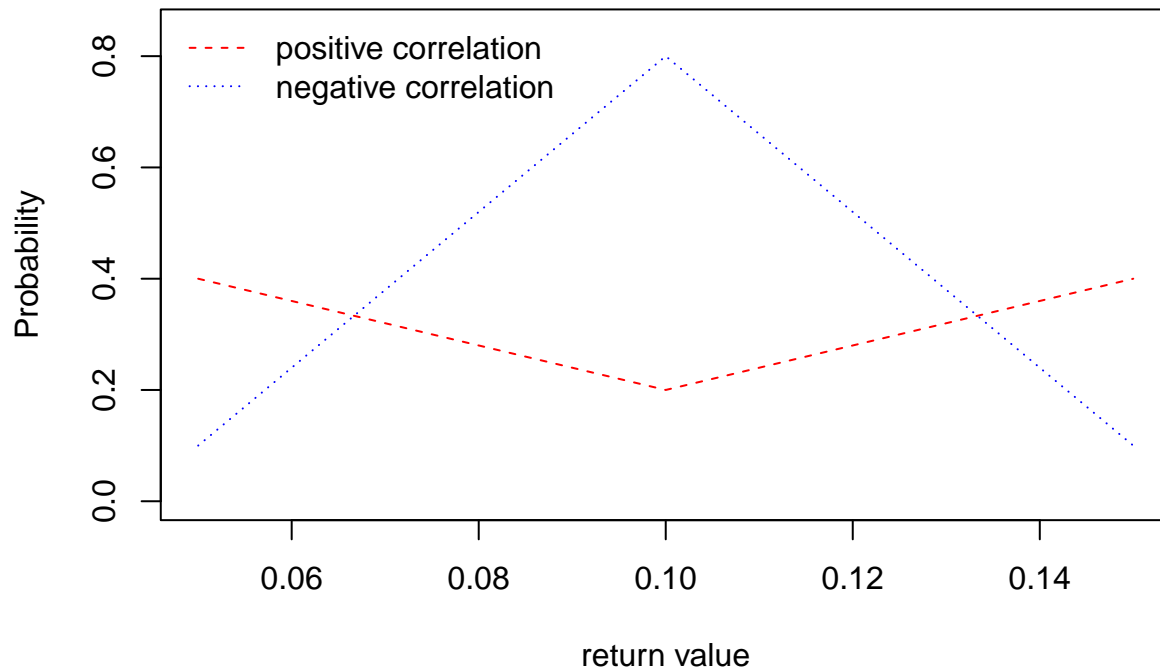
```
## EP,VP: 0.1 5e-04
```

(f)

q: .05, .1, .15  
p(q): .1,.8,.1

```
pv2 = c(.1,.8,.1)
plot(range(xv),c(0,.85),type="n",xlab="return value",ylab="Probability")
lines(xv,pv1,col="red",lty=2)
lines(xv,pv2,col="blue",lty=3)
legend('topleft',legend=c('positive correlation','negative correlation'),lty=c(2,3),col=c('red','blue'))
title(main='distribution of portfolio')
```

## distribution of portfolio



*You can see that the variance is much bigger for red than the blue !!!*

When the returns are negatively correlated the moves tend to cancel each other out.  
If  $X$  is up and  $Y$  is down the average does not move.

In the positively correlated case, they both move the same way and so does the average.

For part (c) note that:

x	y	P(x,y)
0.05	0.05	0.4
0.15	0.05	0.1
0.05	0.15	0.1
0.15	0.15	0.4

so, there is .4 chance both are down (.05) in which case P is .05.

There is .4 chance both are up (.15) in which case P is .15

and a  $.1+.1 = .2$  chance that one is up and the other is down so P is .1.

The slightly tricky thing here is that two of the (x,y) outcomes give you the same value for P so their probability is combined.



## 1.12 Ford and Tesla

In the notes we compute the mean and variance of the portfolio which puts 50% into Ford and 50% into Tesla.

Suppose we put 25% into Ford and 75% into Tesla.

(a)

What are the mean, variance, and standard deviation of the portfolio.

(b)

plot the standard deviations for Ford, Tesla, and the 25/75 portfolio against their means.

## Solution

From the notes we have the 5 numbers we often use to summarize a joint distribution:

- the two means
- the two variances
- the covariance

(a)

```
# Ford and Tesla means
mF = .12
mT = .14

# Ford and Tesla variances
vF = 5.25
vT = 9.76

#covariance
cFT = 3.063

# portfolio weights
w1 = .25
w2 = .75

#mean
mP = w1*mF + w2*mT
cat("mean of portolio is: ",mP,"\n")
```

```
## mean of portolio is: 0.135

#Variance
vP = w1^2 * vF + w2^2 * vT + 2*w1*w2*cFT
cat("variance of portolio is: ",vP,"\n")
```

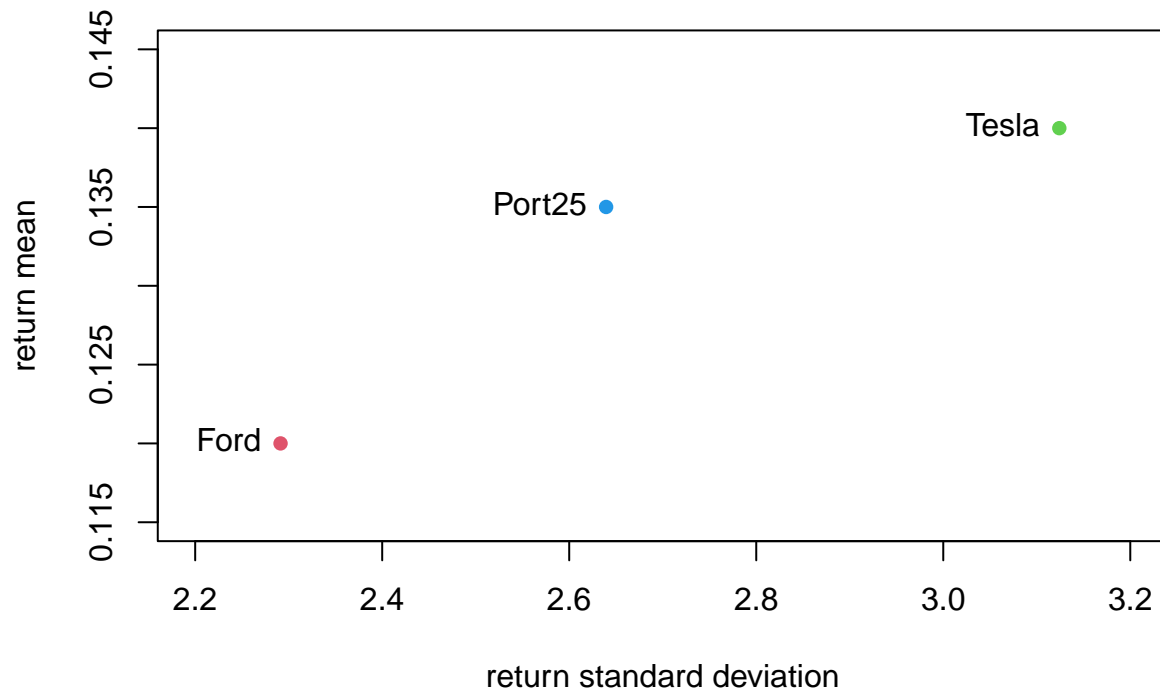
```
## variance of portolio is: 6.96675

#standard deviation
sP = sqrt(vP)
cat("standard deviation of Porfolio is: ", sP, "\n")
```

```
## standard deviation of Porfolio is: 2.63946
```

(b)

```
mv = c(mF,mT,mP)
sF = sqrt(vF)
sT = sqrt(vT)
sv = c(sF,sT,sP)
plot(sv,mv,xlab="return standard deviation",ylab = "return mean",col=2:4,
      xlim=c(2.2,3.2),ylim=c(.115,.145),pch=16)
nms = c("Ford","Tesla","Port25")
text(sv,mv,nms,pos=2)
```



Notice that even though the mean of the portfolio return is closer to Tesla than Ford, the standard deviation is in the middle.

*This is the magic of diversification, even with just two input assets.*

### 1.13 Rolling a Die

Let's roll a die 100 times.

Let  $X_i$  be the random variable representing the outcome for the  $i^{\text{th}}$  die.

We discussed in class that the  $X_i$  are IID.

- (a) What are the mean and variance of each  $X_i$  ?
- (b) What are the mean and variance of the sum of the 100  $X_i$  ?
- (c) What are the mean and variance of the average of the 100  $X_i$  ?

## Solution

(a)

```
x = 1:6
pv = rep(1,6)/6
data.frame(x,pv)
```

```
##   x      pv
## 1 1 0.1666667
## 2 2 0.1666667
## 3 3 0.1666667
## 4 4 0.1666667
## 5 5 0.1666667
## 6 6 0.1666667
```

```
M = sum(x*pv)
V = sum(pv* (x-M)^2)
S = sqrt(V)
cat("one roll, mean, var, sd:",M,V,S,"\n")
```

```
## one roll, mean, var, sd: 3.5 2.916667 1.707825
```

(b)

```
MA = 100*M
VA = 100*V
SA = sqrt(VA)
cat("sum of 100 rolls, mean, var, sd:",MA,VA,SA,"\n")
```

```
## sum of 100 rolls, mean, var, sd: 350 291.6667 17.07825
```

(c)

```
MM = M
VM = V/100
SM = sqrt(VM)
cat("average of 100 rolls, mean, var, sd:",MM,VM,SM,"\n")
```

```
## average of 100 rolls, mean, var, sd: 3.5 0.02916667 0.1707825
```

### 1.14 Thaler Example

In the Thaler example there were 23 executives.

We found that for the CEO, the sharpe ratio (mean/(standard deviation)) was 1.6.

Suppose there are 100 executives instead of 23 so that

$$T = X_1 + X_2 + \dots + X_{100}.$$

What is the sharpe ratio now?

## Solution

```
MT100 = 100 * .5
VT100 = 100 * 2.25
SdT00 = sqrt(VT100)

SharpeT100 = MT100/SdT00

cat("the Sharpe ratio with 100 is: ",SharpeT100)

## the Sharpe ratio with 100 is: 3.333333
```

## 1.15 Working with the Normal Distribution

Recall the Thaler example.

For the case where we have 23 executives and  $T$  is the total total revenue we compute

$$E(T) = 23 * .5 = 11.5, \quad Var(T) = 23 * 2.25 = 51.75, \quad \sigma_T = \sqrt{51.75} = 7.2.$$

We then used the normal approximation to the distribution of  $T$

$$T \sim N(11.5, 7.2^2)$$

Using the normal approximation

(a)

Plot the pdf of  $T$ .

(b)

What is the probability  $P(T < 20)$  ?

(c)

What is the probability  $P(0 < T < 20)$  ?

(d)

What is the probability  $P(30 < T)$  ?

(e)

What is the value  $c$  such that  $P(T < c) = .05$  ?

(f)

Suppose  $T$  turned out to be -15.

To get a sense of how surprising this is, compute the  $z$  value.

What does this tell us?

(g)

Suppose we have 100 executives instead of 23.

Plot the normal pdf representing the approximation distribution.

(h)

Suppose we have 100 executives instead of 23.

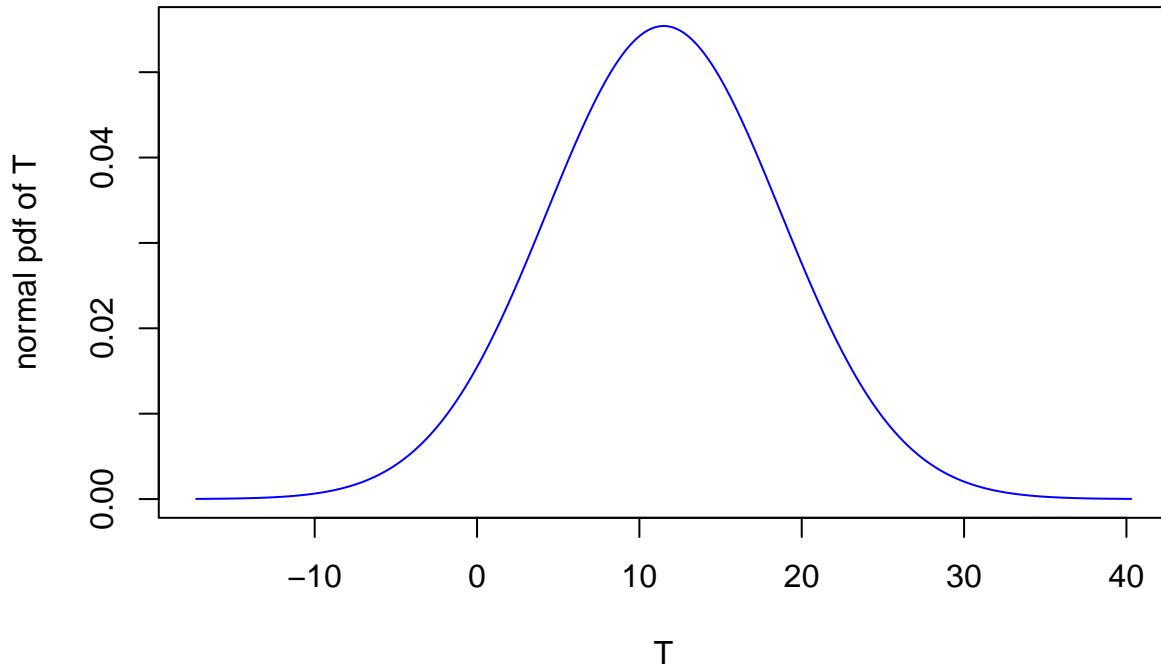
What is the probability  $P(30 < T)$  ?



## Solution

(a)

```
mu = 11.5; sigma = 7.2
Tvals = seq(from = mu - 4*sigma,to = mu+4*sigma, length.out =1000)
Dvals = dnorm(Tvals,mu,sigma)
plot(Tvals,Dvals,xlab="T",ylab="normal pdf of T", type = "l",col="blue")
```



(b)

```
pnorm(20,mu,sigma)
```

```
## [1] 0.8811103
```

(c)

```
pnorm(20,mu,sigma) - pnorm(0,mu,sigma)
```

```
## [1] 0.8260022
```

(d)

```
1-pnorm(30,mu,sigma)
```

```
## [1] 0.005093086
```

(e)

```
cval = qnorm(.05,mu,sigma)
```

```
## check
```

```
ck = pnorm(cval,mu,sigma)
```

```
cat("value is : ", cval, ", check: ",ck,"\n")
```

```
## value is : -0.3429461 , check: 0.05
```

(f)

```
z = (-15-mu)/sigma
cat("z value is: ",z,"\n")
```

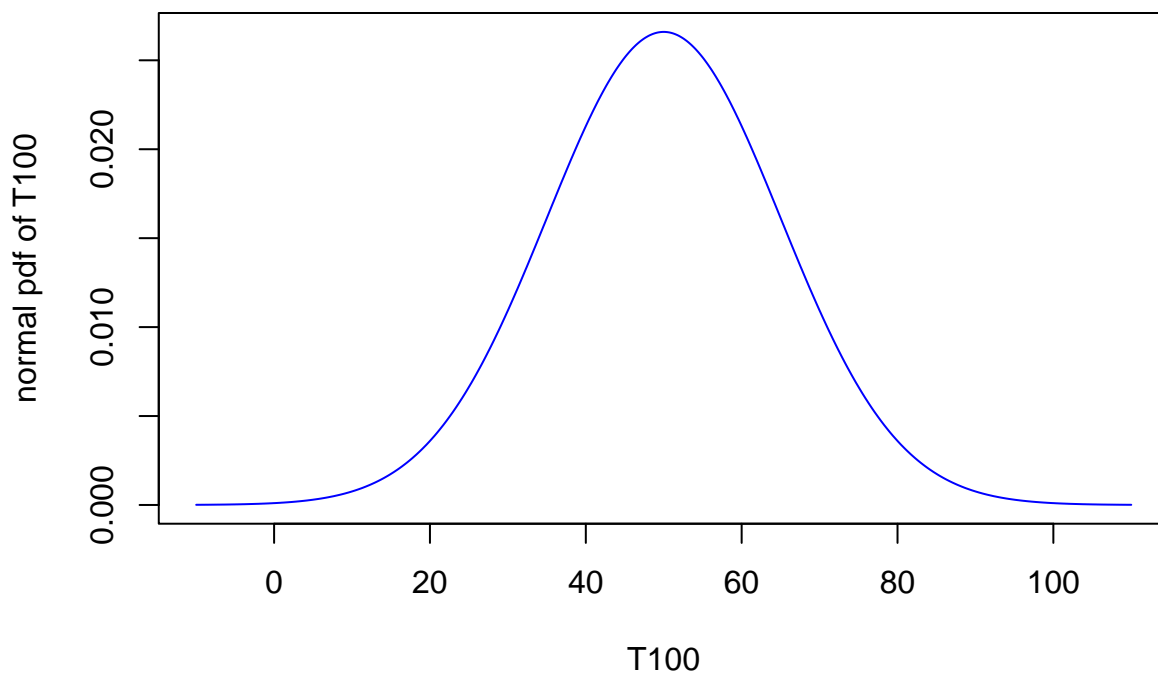
```
## z value is: -3.680556
```

This would be like getting -3.68 from a standard normal which is very unusual.

Or, you could say this is -3.68 standard deviations below the mean.

(g)

```
mu100 = 100*.5; sigma100 = sqrt(100*2.25)
Tvals = seq(from = mu100 - 4*sigma100,to = mu100+4*sigma100, length.out =1000)
Dvals = dnorm(Tvals,mu100,sigma100)
plot(Tvals,Dvals,xlab="T100",ylab="normal pdf of T100", type = "l",col="blue")
```



(h)

```
1-pnorm(30,mu100,sigma100)
```

```
## [1] 0.9087888
```

## 1.16 Portfolios

In the notes we considered the example

$$SP500 \sim N(6, 15^2), \quad Bonds \sim N(2, 5^2), \quad corr(SP500, Bonds) = -0.2.$$

(a)

Verify that for the portfolio that puts 70% into SP500 and 30% into Bonds  $E(P) = 4.8$  and  $\text{Var}(P) = 106.2$ .

(b)

Let  $w_1 + w_2 = 1$ .

Let  $P = w_1 SP500 + w_2 Bonds$ .

So, above we have  $w_1 = .7$ .

Compute the expected return and standard deviation of the portfolio  $P$  for  $w_1 = .1, .3, .5, .7, .9$ .

(c)

Plot the standard deviation versus the means from (b).

## Solution

(a)

```
mS = 6; mB = 2
vS = 15^2; vB = 5^2
covSB = -.2*sqrt(vS)*sqrt(vB)
w1 = .7; w2 = 1-w1
ep = w1*mS + w2*mB
vp = w1^2 * vS + w2^2 * vB + 2*w1*w2*covSB
cat("EP, VP: ", ep, vp, "\n")
```

```
## EP, VP: 4.8 106.2
```

(b)

```
wv = c(.1, .3, .5, .7, .9)
np = length(wv)
ev = rep(0, np)
vv = rep(0, np)
for(i in 1:np) {
  w1 = wv[i]; w2 = 1-w1
  ev[i] = w1*mS + w2*mB
  vv[i] = w1^2 * vS + w2^2 * vB + 2*w1*w2*covSB
}
print(ev)
```

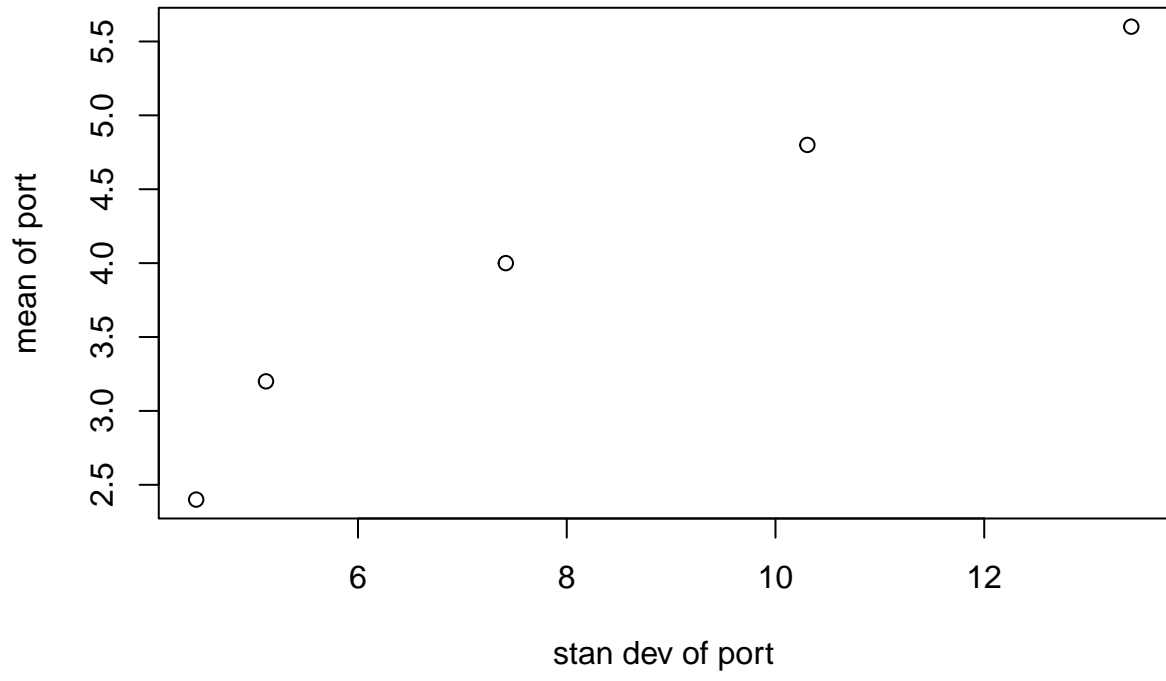
```
## [1] 2.4 3.2 4.0 4.8 5.6
```

```
print(vv)
```

```
## [1] 19.8 26.2 55.0 106.2 179.8
```

(c)

```
sv = sqrt(vv)
plot(sv, ev, xlab = "stan dev of port", ylab = "mean of port")
```



## 1.17 Uniform Remnant

Suppose you manage a process that cuts cloth.

For every cut, there is a wasted remnant.

If  $R$  is the random variable representing the remnant, then you have decided

$$R \sim \text{Uniform}(0, .5), \text{ (inches)}$$

That is, \* any value between 0 and .5 is equally likely \*.

(a)

Plot the pdf of  $R$ .

(b)

What is  $P(R > .3)$  ?

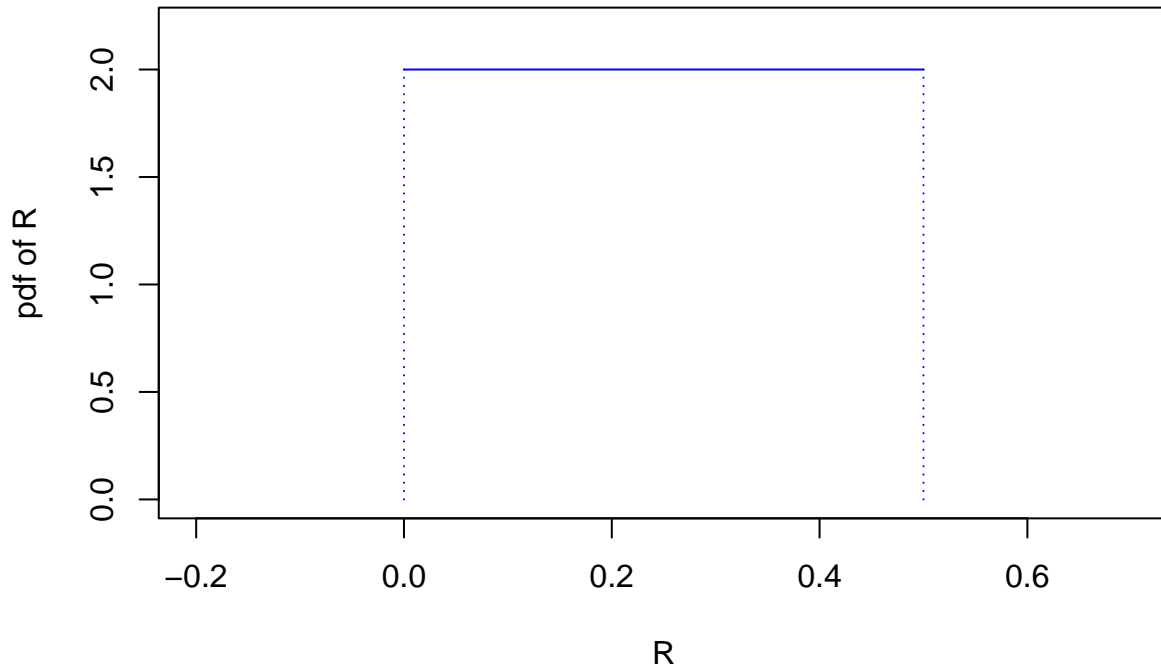
(c)

What is  $E(R)$  ?

Solution

(a)

```
plot(c(-.2,.7),c(0,2.2),xlab = "R",ylab="pdf of R",type="n")
lines(c(0,.5),c(2,2),col="blue")
lines(c(0,0),c(0,2),col="blue",lty=3)
lines(c(.5,.5),c(0,2),col="blue",lty=3)
```



(b)

```
(.5-.3)*2
```

```
## [1] 0.4
```

(c)

```
(.5-0)/2.0
```

```
## [1] 0.25
```

## 1.18 Monte Carlo to check Normal Approximation

Note: you may find the problem considerably harder than most!!

Lets' use the Thaler example again and check the normal approximation using Monte Carlo.

We can use the `sample` function to get IID draws for the distribution for each executive, where the outcome is -1 or 2 with probabilities .5 and .5.

First let see how to use `sample` to get IID draws.

```
pv = c(.5, .5)
vv = c(-1, 2)
set.seed(99)
nd = 10000
temp = sample(vv, size = nd, replace=TRUE, prob=pv)
table(temp)/nd
```

```
## temp
##      -1      2
## 0.4932 0.5068
```

OK, now I want to repeatedly get 23 IID draws and take the sum. I will do this `nd = 5000` times.

```
nexec = 23
nd = 20000
set.seed(14)
tvec = rep(0, nd)
for(i in 1:nexec) {
  tvec[i] = sum(sample(vv, size=nexec, replace=TRUE, prob=pv))
}
## check the mean and stan dev
mcheck = mean(tvec)
sdcheck = sd(tvec)
cat("monte carlo mean and sd: ", mcheck, sdcheck, "\n")

## monte carlo mean and sd:  11.53285 7.165458
```

(a)

Compute  $P(T > 0)$  using the Monte Carlo draws in `tvec` and using the normal approximation, do they match up?

(b)

How does the histogram of the values in `tvec` compare to the approximate normal density ?



*Solution*

(a)

```
mcest = sum(tvec>0)/length(tvec)
norapprox = 1 - pnorm(0,11.5,7.2)
cat("monte carlo, normal: ",mcest,norapprox,"\n")
```

```
## monte carlo, normal: 0.9549 0.9448919
```

(b)

```
hist(tvec,prob=TRUE,nclass=6,ylim=c(0,.06))
tv = seq(from = -15,to=40,length.out = 1000)
lines(tv,dnorm(tv,11.5,7.2),col="red",type="l",lty=2)
```

**Histogram of tvec**

