A Simple Example of a Markov Chain

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A Simple Two State Chain

Let's consider a simple two state Markov chain with transition matrix

$$P = \begin{bmatrix} .5 & .5 \\ .25 & .75 \end{bmatrix}$$

where,

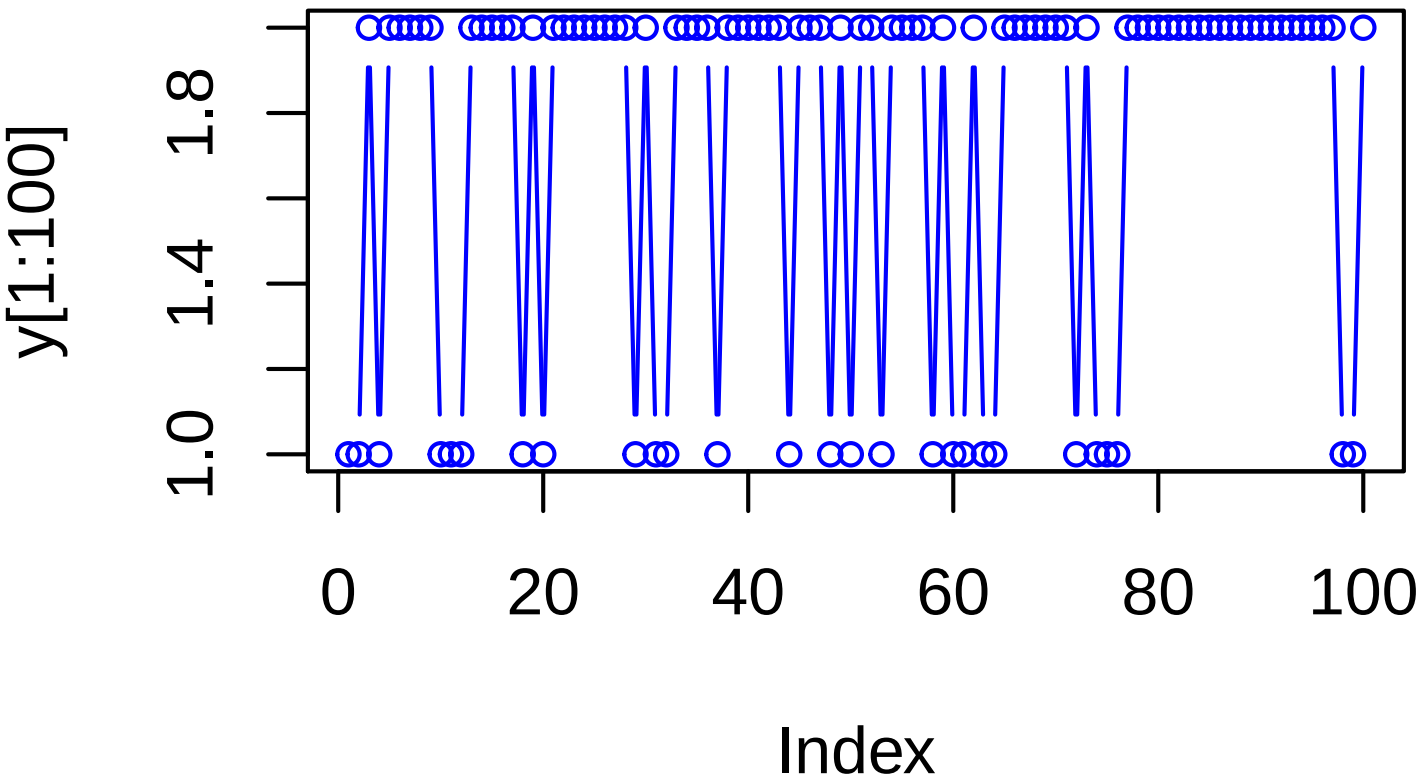
$$P_{ji} = Prob(Y_t = i | Y_{t-1} = j)$$

To simulate the chain:

```
P = matrix(c(.5,.25,.5,.75),nrow=2)
nd=5000
y = rep(0,nd)
y[1]=1
for(i in 2:nd) {
   y[i] = sample(1:2,1,prob=P[y[i-1],])
}
```

Let's plot the first 100 draws:

```
plot(y[1:100],type="b",pch=1,col="blue",cex=.75)
```



What is the stationary distribution?

```
eg = eigen(t(P))
print(eg)
```

```
## $values
## [1] 1.00 0.25
##
## $vectors
##            [,1]       [,2]
## [1,] -0.4472136 -0.7071068
## [2,] -0.8944272  0.7071068
```

```
pi = eg$vectors[,1]; pi = pi/sum(pi)
print(pi)
```

```
## [1] 0.3333333 0.6666667
```

And how does the stationary distribution relate to the draws?

```
print(table(y)/length(y))
```

```
## y
##     1     2
## 0.326 0.674
```

Let $f(y) = y^2$.

Then $E_\pi(f(Y)) = 1/3 + 2 * 4/3 = 3$.

Which we can get by MCMC (Markov Chain Monte Carlo) using

```
fv = y^2
print(mean(fv))
```

```
## [1] 3.022
```

Note:

```
Pt=P
for(i in 1:5000) {
   Pt = Pt %*% P
}
print(Pt)
```

```
##           [,1]      [,2]
## [1,] 0.3333333 0.6666667
## [2,] 0.3333333 0.6666667
```