Mixture Model

$$p(p|\alpha)\, p(\mu|\bar{\mu}, \sigma_\mu^2)\, p(\sigma|\nu, \lambda)$$
$$p(I|p)\, p(y|\mu, \sigma, I)$$

$$p = (p_1, p_2, \ldots p_k) \sim \text{Dirichlet}(\alpha)$$

$$\mu = (\mu_1, \mu_2, \cdots \mu_k) \sim N(\bar{\mu}, \sigma_\mu^2) \quad \text{iid}$$

$$\sigma = (\sigma_1, \sigma_2, \ldots \sigma_k) \sim \frac{\nu\lambda}{\chi^2_\nu} \quad \text{iid}$$

$$I = (I_1, \ldots I_n)$$
$$p(I_i = j) = p_j$$

$$y = (y_1, \ldots y_n) \qquad y_i | I_i = j \sim N(\mu_j, \sigma_j^2)$$

independent

Suppress Hyperparameters

$$p(p, I, \mu, \sigma, y) =$$

$$p(p)\, p(I \mid p)\, p(\mu)\, p(\sigma)\, p(y \mid \mu, \sigma, I)$$

p conditional

$$p(p \mid I, \mu, \sigma, y) \propto p(p)\, p(I \mid p)$$

$\sim$ simple Dirichlet

μ conditional

$$p(\mu \mid p, I, \sigma, y) \propto p(\mu)\, p(y \mid \mu, \sigma, I)$$

$$\propto \left[\prod_j p(\mu_j)\right] \left[\prod_j \prod_{i: I_i = j} p(y_i \mid \mu_j, \sigma_j)\right]$$

$$\propto \prod_j \left[p(\mu_j) \prod_{i: I_i = j} p(y_i \mid \mu_j, \sigma_j)\right]$$

- each μ_j is an independent normal

σ conditional

Each σ_j is an independent inverted χ^2

I conditional

$$p(I \mid p, \mu, \sigma, y) \propto p(I \mid p)\, p(y \mid \mu, \sigma, I)$$

$$= \prod_i p(I_i \mid p) \prod_i p(y_i \mid \mu, \sigma, I_i)$$

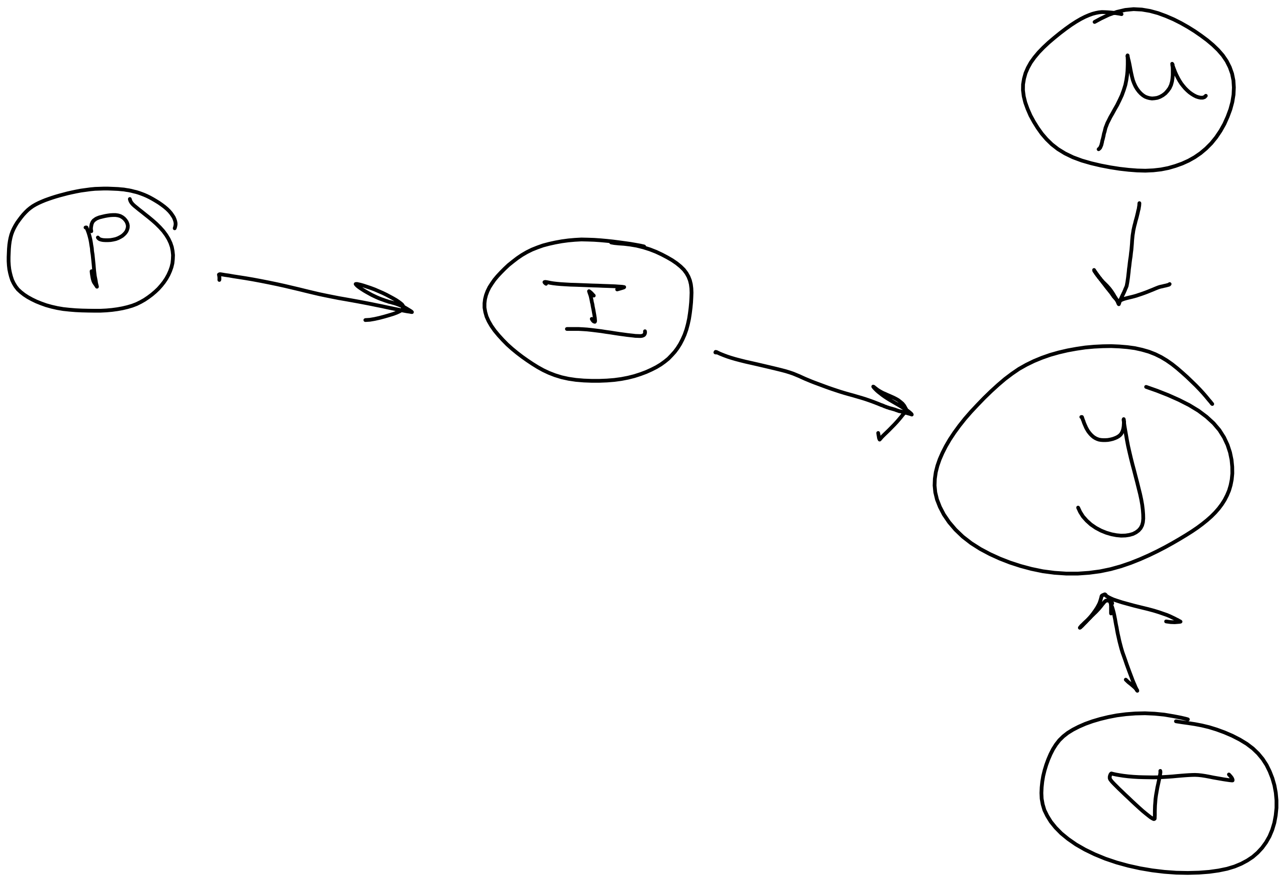
$$= \prod_i p(I_i \mid p)\, p(y_i \mid \mu, \sigma, I_i)$$

I_i 's are independent!

$$p(y_i \mid \mu, \sigma, I_i = j) \sim n(y_i \mid \mu_j, \sigma_j)$$

$$p(I_i = j \mid y_i, \mu, \sigma, p)$$

$$\propto \quad p_j \, n(y_i \mid \mu_j, \sigma_j) \, .$$



DAG of model

Directed acyclic graph.