XP83 Statistics Final

Fall, 2012

Name: SOLUTIONS

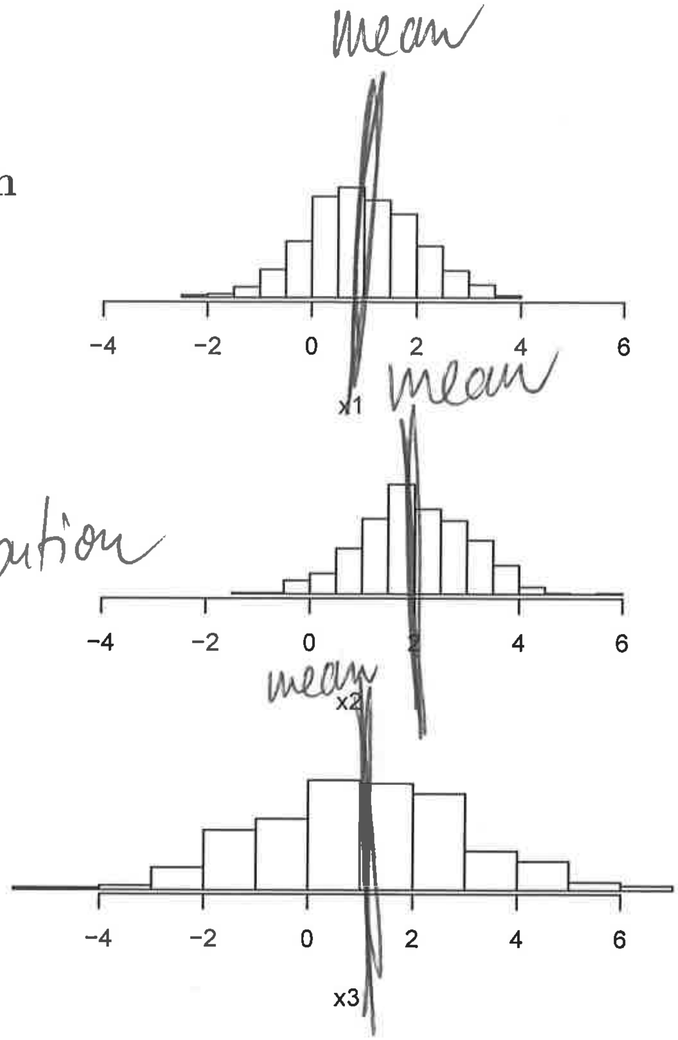
I pledge my honor that I have not violated the Honor Code:

Note:

- You have three hours.
- You may use a pen and a calculator.
- A formula sheet has been provided.
- There are 12 questions.
- Each part of each question is worth 2 points.

1 Question

Mean: Where is the distribution centered? (for well-behaved distributions)



The three histograms above depict observations on the variables x_1, x_2, and x_3.

Each has average either 1 or 2 and standard deviation either 1 or 2.

(1.1) The mean of x_1 is <u>1</u>. The standard deviation of x_1 is <u>1</u>.

(1.2) The mean of x_2 is <u>2</u>. The standard deviation of x_2 is <u>1</u>.

(1.3) The mean of x_3 is <u>1</u>. The standard deviation of x_3 is <u>2</u>.

X_3 is more dispersed ("spread out") than X_1 & X_2

(1.4) What is the variance of x_2 ?

$V(X_2) = \sigma^2_{x_2} = (1)^2 = 1$ variance is st. dev. squared

(1.5) Give an interval which should contain roughly 95% of the x_1 values.

Confidence interval for 95% is mean $\pm$ 2 × stdev

$1 \pm 2 \times 1 = 1 \pm 2 = [-1; 3]$

2 Question

In the countries (conret.xls.txt) data, the usa variables tells us what returns were for a series on months on a portfolio made up of American assets.

The average usa return is .01346.
The standard deviation of usa returns is .0333.

usa is the returns you would have gotten if you put all your money into the american portfolio.

Suppose (unrealistically) that there was an investment available that would give you a return of .002 for sure each month. This is a "riskless" asset.

Suppose instead of putting all your money in usa, you put 60% in the riskless asset and 40% in usa.

Your returns for each month would have been

$$rp = .6\,(.002) + .4\,\text{usa}$$

where usa means the usa return for a given month.

In General: If $y = a + bx$

$\Rightarrow \bar{y} = a + b\,\bar{x}$; $S_y = |b|\, S_x$

2.1

If you had done this, what would be the mean of your returns?

$\overline{rp} = (.6) \times (.002) + (.4) \times \overline{usa}$

$= (.6) \times (.002) + (.4) \times (.01346) = 0.006584$

2.2

If you had done this, what would be the standard deviation of your returns?

$S_{rp} = |.4| \times S_{usa}$

$= (.4) \times (.0333) = 0.01332$

3 Question

Suppose the distribution of the random variable X is given by the following table

```
x          p(x)
--         ---
1           .25
2           .5
3           ?? .25
```

3.1

What is $P(X = 3)$?

All probs have to sum to 1

So $P(X=3) = 1 - .25 - .5 = \underline{\underline{.25}}$

3.2

What is $P(X < 3)$?

$P(X<3) = P(X=1) + P(X=2) = .25 + .5 = \underline{\underline{.75}}$

3.3

What is $E(X)$?

$E(X) = \sum_i x_i p(x_i) = 1 \times (.25) + 2 \times (.5) + 3 \times (.25)$

$= \underline{\underline{2}}$

3.4

What is $Var(X)$?

$V(X) = \sum_i p(x_i)(X_i - E(X))^2 = .25 \times (1-2)^2 + .5(2-2)^2$

$+ .25(3-2)^2 = \underline{\underline{.5}}$

3.5

What is σ_X.

$\sigma_X = \sqrt{V(X)} = \sqrt{.5} = \underline{\underline{.7071}}$

4 Question

The table below gives the joint distribution of X and Y.

joint probs

		X = 0	X = 1	
Y	0	.36	.24	.6
	1	.24	.16	.4
		.6	.4	

marginal of Y

marginal of X

4.1

What is $P(X = 0, Y = 1)$?

$= .24$

4.2

What is $P(X = 0)$?

$= P(X=0, Y=0) + P(X=0, Y=1) = .36 + .24 = .6$

4.3

What is $P(X = 0 \mid Y = 1)$?

$$= \frac{P(X=0, Y=1)}{P(Y=1)} = \frac{.24}{.4} = .6$$

$P(Y=1) = P(Y=1, X=0) + P(Y=1, X=1) = .4$

4.4

Are X and Y independent?

$P(X=1) \times P(Y=1) = (.4)\times(.4) = .16$

$P(X=1, Y=1) = .16$ same $\Rightarrow$ YES!

4.5

Is X a Bernoulli random variable?

X is 0 or 1 $\Rightarrow$ YES

4.6

What is $E(X)$?

For Bernoulli: $E(X) = p = P(X=1) = .4$

or $E(X) = 0\times(.6) + 1\times(.4) = .4$

4.7

What is $Var(X)$?

For Bernoulli: $V(X) = p\times(1-p) = .4\times(1-.4)$

$= .24$

or $V(X) = .6\times(0-.4)^2 + .4(1-.4)^2 = .24$

4.8

Are X and Y iid?

iid is independent and identically distrib.

Yes, we checked that above

X	$P(X)$
0	.6
1	.4

~~P(Y) p~~

Y	$P(Y)$
0	.6
1	.4

$\Rightarrow$ Yes

YES!

4.9

What is the covariance between X and Y?

$Cov(X,Y) = E(XY) - E(X)E(Y)$

4.9)

$X \quad Y \quad P(x,y)$

$$
\begin{aligned}
E(XY) = {} & 0 \times 0 \times .36 \\
& + 1 \times 0 \times .24 \\
& + 0 \times 1 \times .24 \\
& + 1 \times 1 \times .16 \\
\hline
& = 0.16
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{cov}(X,Y) &= E(XY) - E(X)E(Y) \\
&= 0.16 - (0.4)(0.4) = 0
\end{aligned}
$$

---

or X & Y are independent (from 4.4)

$\Rightarrow \operatorname{cov}(X,Y) = 0$

independence implies $\operatorname{cov}(X,Y)=0$

5 Question

$\sigma_{R2} = \sqrt{.01} = .1$

Suppose

$$\overset{\mu_{R_1}\ \sigma^2_{R_1}}{} \qquad \overset{\mu_{R_2}\ \sigma^2_{R_2}}{}$$

$$R_1 \sim N(.2, .01), \quad R_2 \sim N(.1, .01).$$

$\sigma_{R_1} = \sqrt{.01} = .1$

The correlation between R_1 and R_2 is .7. $\rho_{R_1 R_2}$

Let

$$P = .4\, R_1 + .6\, R_2.$$

5.1

What is the covariance between R_1 and R_2?

$\sigma_{R_1 R_2} = \rho_{R_1 R_2} \times \sigma_{R_1} \times \sigma_{R_2} = .7 \times .\phi \times .1 = \underline{\underline{.007}}$

5.2

What is $E(P)$?

$E(P) = .4\, E(R_1) + .6\, E(R_2)$

$= .4 \times .2 + .6 \times .1 = \underline{\underline{.14}}$

5.3

What is $Var(P)$?

$Var(P) = (.4)^2\, V(R_1) + (.6)^2\, V(R_2)$

$+ 2 \times (.4) \times (.6) \times \sigma_{R_1 R_2}$

$= (.4)^2 \times .01 + (.6)^2 \times .01 + 2 \times (.4) \times (.6) \times (.007)$

$= \underline{\underline{.00856}}$

6 Question

Let R denote the uncertain return on an asset next period.
Our uncertainty is represented by

$$R \sim N(.1, .01).$$

6.1

What is $E(R)$?

$.1$

6.2

What is $Var(R)$?

~~$\sqrt{.01} = .1$~~ $.01$

6.3

What is σ_R ?

$\sqrt{.01} = .1$

6.4

What is $P(R > 0)$?

$P(R>0) = P(R>\mu) = 50\%$

Working with returns makes us work small numbers. In some cases, a change of a half of one percent is a big deal. If mortgage rates go from 4.5% to 4.0% that matters and that is a change from .045 to .04. Often people work in terms of *basis points*. 100 basis points = 1%.

So, if we want to represent a return as a "percent" we would multiply by 100 (.1 is 10%) and if we want to represent a return in basis points we multiply by 10,000 (.01 is 1% is 100 basis points).
In basis points,

$$B = 10000\, R$$

6.5

What is $E(B)$?

$$E(B) = 10{,}000 \times E(R) = 10{,}000 \times (.1)$$
$$= \underline{\underline{1{,}000}}$$

6.6

What is σ_B?

$$\sigma_B = |10{,}000| \times \sigma_R$$

$$= 10{,}000 \times .1 = \underline{\underline{1{,}000}}$$

Suppose R_1 and R_2 are iid $N(.1,.01)$.
Let $B_1 = 10000R_1$ and $B_2 = 10000R_2$
(the B's are the R's expressed in basis points).

Suppose we are interested in the difference in the returns expresses as basis points.

Let

$$D = B_1 - B_2.$$

6.7

What is $E(D)$?

$$E(D) = 10{,}000\,E(R_1) - 10{,}000\,E(R_2)$$
$$= 10{,}000 \times (.1) - 10{,}000 \times (.1) = \underline{\underline{0}}$$

6.8

What is $Var(D)$?

$$V(D) = V(B_1) + V(B_2) = (1{,}000)^2 \times 2$$
$$= \underline{\underline{2{,}000{,}000}}$$

6.9

Give an interval such that there is a 95% chance D will end up being in it.

$$E(D) \pm 2 \times \sqrt{V(D)}$$

$$= 0 \pm 2 \times \sqrt{2{,}000{,}000}$$

$$= \underline{\underline{0 \pm 2828}}$$