Modeling with IID Normal Draws
Homework Problems
Homework Solutions

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3.1. Problem: Australian Return Data

(a)

Do the time series plot and histogram of the "australia" returns from the countries data (conret.csv). These are returns on a portfolio of Australian equities (stocks).
Does it look reasonable to model the returns as iid draws from a normal distribution?

Note: the "time series plot" is just i versus x_i for values x_i.
The is also called the sequence plot. We have already used this plot a lot!!

In Excel: Select the column of Australian returns and then go to /Insert and then play around to find the plot type you like the best.
If you just do scatterplot, it seems to look ok.

In R:

```
cdat = read.csv("http://www.rob-mcculloch.org/data/conret.csv")
plot(cdat$australia,type="b",xlab="year number",
          ylab="Australia Returns",col="blue")
```

(b)

Let μ be the sample average of the australian returns and σ be the sample standard deviation.

If we go ahead and model our returns as iid normal and use these values for μ and σ, what is the probability of a negative return?

(c)
What is the "z" value corresponding to $r = 0$
$(z = (r - mu)/sigma)$?

Check that $P(Z < z)$ gives you the same number you got in part (b), where $Z \sim N(0, 1)$.

(d)
z each australian return.
Do the histogram of the z values.
Does it kind of look like the standard normal pdf?!!

In Excel you could copy the formula
=(B2 - AVERAGE($B2:$B108))/STDEV.S($B2:$B108)
in cells 2:108 in column Y (or any empty column).
Remember to copy a formula:

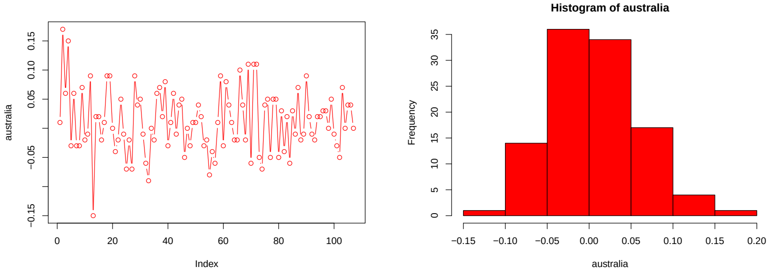
- click in the top cell (e.g. Y2))
- shift/click in the bottom cell (e.g. Y108))
- type in the formula
- enter Cntl/return.

In R:

```
za = (cdat$australia - mean(cdat$australia))/sd(cdat$australia)
```

Solution

(a)



Maybe one negative outlier, but overall looks pretty normal.

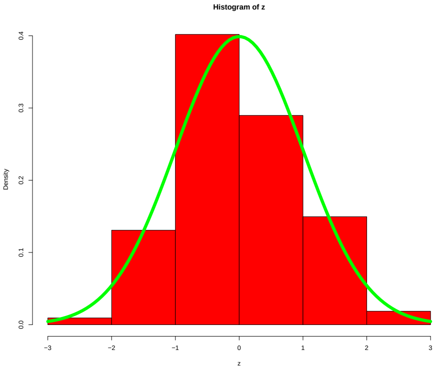
(b)

mu,sigma: 0.01224299 0.05413659
prob of negative return: 0.4105424.

(c)

z = (0-0.01224299)/0.05413659 = -0.22615
F(-0.22615) = 0.4105424

(d)



Looks pretty good!!

3.2. Problem: Unusual Belgian Return?

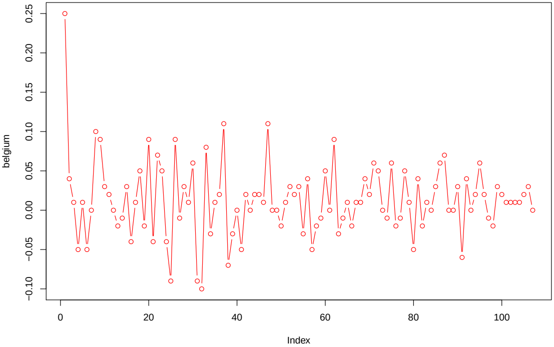
Do the time series plot of the "belgium" returns form conret.csv.

Notice that the first return seems like an "outlier".

Let μ be the sample average of the returns, excluding the first one. Let σ be the sample standard deviation of the returns, excluding the first one.

What is the z value for the first return?
Is it more unusual than Gretzky?

Solution



First one really sticks out!!

mu,sigma: 0.01103774 0.0416317

$z = (.25\text{-}0.01103774)/0.0416317 = 5.739911$

Much more unusual than Gretzky!!

3.3. Problem: The Random Walk Model

Get the data Price-level.csv from the webpage.
There is just one column called `Price`.
The data are the monthly price of an asset for 500 months (about 41 years).

(a)

Do a time series plot of the prices.
Could they be modeled as iid ?

(b)
Compute the price differences.

If the prices are denoted by P_t, then the differences are
$D_t = P_t - P_{t-1}, t = 2, 3, \ldots, n$.

For example, the first three prices are 0.000000, -1.748190, and 4.707081
so the first two differences are
-1.748190-0.000000 = -1.74819 and
4.707081 - (-1.748190) = 6.455271

```
> Price[1:4]
[1]  0.000000 -1.748190  4.707081  8.905969
> -1.748190-0.000000
[1] -1.74819
> 4.707081 - (-1.748190)
[1] 6.455271
```

Note that in R we could get the differences by

```
> ii = 2:n
> D = Price[ii]-Price[ii-1]
> print(D[1:3])
[1] -1.748190  6.455271  4.198888
```

Do a time-series plot and a histogram of the differences.

Do the differences look iid normal?

(c)
Give a distribution describing what the next price difference will be.

That is, our last observed price is $P_{500} = 609.298161$.
So the next difference is $P_{501} - 609.298161$ which is a number we are unsure about because we don't know the next price P_{501}.

(d)
Give a distribution describing what the next price (P_{501}) will be.

Our model is:

$$D_t = P_t - P_{t-1} \sim N(\mu, \sigma^2).$$

This is the very famous *random walk model* which we can express as

$$P_t = P_{t-1} + D_t$$

The next price, is the current price plus a "random" increment. In this problem we modeled the increment as iid normal.

Solution

Solution.