

Linear Combinations, Covariance and Correlation

Homework Problems

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1. Mean and Variance of a Linear Function
 - 1.1. Problem: Invest-100
 - 1.2. Problem: Uniform Return
2. Covariance and Correlation for RVs
 - 2.1. Problem: Covariance and Correlation
3. Mean and Variance of a Linear Combination
 - 3.1. Problem: Portfolio Return, Three Risky Assets
 - 3.2. Problem: Sum and Average of IID

1.1. Problem: Invest-100

Suppose you invest \$100 in an asset.

The return on the asset is the random variable R .

$$E(R) = .1.$$

$$\text{Var}(R) = .01.$$

Let W be your end of period wealth so that $W = (1 + R)100$.

(a)

What is $E(W)$?

(b)

What is $\text{Var}(W)$?

(c)

Suppose R is normal.

Any linear function of a normal random variable is also normal.

Once you know an RV is normal, all you need is its mean and variance and you know the distribution!

What is the distribution of W ?

(d)

What is $\text{Prob}(W > 120)$?

1.2. Problem: Uniform Return

Let U be the uniform random variable which is equally likely to be any value between 0 and 1.

Let R denote a return about which you are uncertain. Suppose you believe that R is equally likely to be any value between $-.1$ and $.2$.

(a) What does the pdf of R look like?

(b) Write R as a linear function of U .

(c)

We know $E(U) = .5$ and $Var(U) = 1/12$.

What is $E(R)$?

What is $Var(R)$?

2.1. Problem: Covariance and Correlation

Suppose V and W are discrete RVs with joint distribution give by:

		W	
		5	15
V	5	.05	.05
	15	.45	.45

Suppose Y and X are discrete RVs with joint distribution give by:

		X	
		5	15
Y	5	.45	.05
	15	.05	.45

- (a) Compute the covariance and correlation between X and Y .
- (b) Compute the covariance and correlation between W and V .
- (c) Are X and Y independent?
- (d) Are W and V independent?
- (e)
Are X and Y identically distributed?
Are X and Y iid ?

3.1. Problem: Portfolio Return, Three Risky Assets

Let R_f denote the return on a riskless asset (you get return R_f for sure!).

Suppose $R_f = .02$.

Suppose R_1 , R_2 , and R_3 are random variables representing returns on three risky assets.

$E(R_1) = .05$, $E(R_2) = .1$, $E(R_3) = .15$.

For each risky asset, the standard deviation equals the mean.

The correlation between R_1 and R_2 is .5 and R_3 is independent of R_1 and R_2 .

Suppose $P_1 = .4 R_f + .6 R_3$.

(a) What is $E(P_1)$?

(b) What is $Var(P_1)$?

(c) What is the correlation between P_1 and R_3 ?

Suppose $P_2 = .2 R_f + .4 R_1 + .4 R_2$.

(d) What is $E(P_2)$?

(e) What is $Var(P_2)$?

(f) What is the correlation between P_1 and P_2 ?

3.2. Problem: Sum and Average of IID

Suppose we can play a game which gives a random payout.

Let W_i denote the payout on the i^{th} play of the game.

The W_i are iid, with $E(W_i) = 0$, $\text{Var}(W_i) = 10$.

(a)

Suppose there is no cover charge and you play the game the first two times so that your total winnings are

$$T = W_1 + W_2.$$

What are $E(T)$ and $\text{Var}(T)$?

(b)

Suppose you are interested in your average winnings:

$$\bar{W} = \frac{W_1 + W_2}{2} = \frac{1}{2}W_1 + \frac{1}{2}W_2.$$

What are $E(\bar{W})$ and $\text{Var}(\bar{W})$?

(c)

Now suppose you play 100 times.

$$T = W_1 + W_2 + \dots + W_{100} = \sum_{i=1}^{100} W_i.$$

$$\bar{W} = \frac{1}{100}(W_1 + W_2 + \dots + W_{100}) = \frac{1}{100} \sum_{i=1}^{100} W_i.$$

What are $E(T)$ and $Var(T)$?

What are $E(\bar{W})$ and $Var(\bar{W})$?